Department of Mathematics
PG Entrance Test 2014

You are required to attempt 35 questions – 5 questions from Section A and 30 questions from Section B. There are a total of 60 questions in Section B and you must attempt only 30 of these. If you attempt more than 30, only the first 30 will be evaluated. All questions carry 2 marks each.

Section A.

Do all five questions from this section.

1. If $f(x) = \frac{x^2 - 4}{x - 2}$ and $g(x) = x + 2$, then
   a. $f = g$
   b. $f \neq g$
   c. $f = -g$
   d. $g = -f$

2. In a group of 250 persons, 210 can speak Kashmiri and 120 can speak English. How many of them can speak both Kashmiri and English?
   a. 80
   b. 330
   c. 90
   d. 10

3. If A and B are mutually exclusive events, then $P(A \cap B) =$
   a. Not defined
   b. 0
   c. 1
   d. None of these

4. The slope of the tangent line to the graph of $f(x) = 3x^2 - 7x + 2^5$ at $x = -1$ is
   a. 6
   b. -6
   c. -13
   d. None of these

5. $\int_0^3 f(x)dx$ represents the
   a. Area under f for $x < 3$
   b. Area under f between 0 and 3
   c. The antiderivative of f at 3
   d. The antiderivative of f at 0

Section B

Attempt only 30 out of the 60 questions in this section. If you attempt more than 30, only the first 30 will be evaluated.

1. Which of the following always holds
    a. Every convergent sequence is bounded
    b. Every bounded sequence is convergent
    c. Both a. and b.
    d. None of these

2. If for two values of X, $A.M = 25$, $H.M = 9$, then the $G.M$ is
    a. 17
    b. 15
    c. 5.83
    d. None of the above

3. If $x^2 - 2x\cos\theta + 1 = 0$, then the value of $x^n + x^{-n}$ is equal to
    a. $2^n \cos\theta$
    b. $2^n \cos^n \theta$
    c. $2\cos n\theta$
    d. $2\cos^n \theta$

4. If each value of a series is divided by 5, its coefficient of variation is reduced by
    a. 0%
    b. 5%
    c. 10%
    d. 20%

5. The image of a closed interval under a continuous function is:
    a. A closed interval
    b. An open interval
    c. Could be either open or closed
    d. Is neither open nor closed.

6. Given that $P(A) = \frac{1}{3}, P(B) = \frac{1}{4}, P(A|B) = \frac{1}{6}$, the probability $P(B|\bar{A})$ is equal to
    a. 1/16
    b. 15/24
    c. 15/16
    d. 5/16

7. For any infinite subset of $\mathbb{R}$ a limit point is always guaranteed if
    a. The subset is open
    b. The subset is bounded
    c. The subset is countable
    d. None of the above

8. A consignment of 15 record players contains 4 defectives. The record players are selected at random one by one and examined. The ones examined are not put back. The probability that the 9th piece examined is the last defective one is
    a. 24/455

b. 8/195
c. 96/195
d. None of these`

9. The integral $\int_0^{\infty} \frac{dx}{x^p}$ diverges for
   a. $p > 1$
   b. $p < 1$
   c. $p = 1$
   d. All of the above

10. If X and Y are two Poisson variates such that X~P(1) and Y~P(2), the probability P(X+Y<3) is
    a. e^{-3}
    b. $3e^{-3}$)
    c. $8.5e^{-3}$
    d. $4e^{-3}$

11. The function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = (x^2 + 1)^{35} \quad \forall\, x \in \mathbb{R}$ is
    a. One-to-one but not onto
    b. Onto but not one-to-one
    c. Neither one-to-one nor onto
    d. Both one-to-one and onto

12. The moment generating function for geometric distribution with parameter p is
    a. $\mathrm{p}(1 - \mathrm{qe}^{\mathrm{t}})$
    b. $\mathrm{p}(1 - \mathrm{qe}^{\mathrm{it}})$
    c. $\mathrm{p}/(1 - \mathrm{qe}^{\mathrm{it}})$
    d. $\mathrm{p}/(1 - \mathrm{qe}^{\mathrm{t}})$

13. The series $\sum(1 + 1/n)^{-n^2}$ is
    a. Convergent
    b. Divergent
    c. Oscillating
    d. Unbounded

14. The conditional distribution of Y given X for the joint p.d.f, $f(x, y) = 3 - x - y$ for $0 \leq x, y \leq 1$ is
    a. $f(y/x) = \frac{3-x-y}{\frac{5}{2}-y}$
    b. $f(y/x) = \frac{3-x-y}{\frac{5}{2}-x}$
    c. $f(y/x) = \frac{\frac{5}{2}-y}{\frac{5}{2}-x}$
    d. None of these

15. The values of $a \; and \; b$ for which the function $f(x) = \begin{cases} 2x + 1, & x \leq 1 \\ ax^2 + b, & 1 < x < 3 \\ 5x + 2a, & x \geq 3 \end{cases}$ is continuous everywhere are
    a. $a = 2,\ b = 1$
    b. $a = 1,\ b = 2$
    c. $a = 3,\ b = 2$
    d. $a = 2,\ b = 3$

16. If $X \sim N(\mu, \sigma^2)$, the points of inflection of normal distribution curve are
    a. $\pm\mu$
    b. $\mu \pm \sigma$
    c. $\sigma \pm \mu$
    d. $\frac{1}{2}(\mu \pm \sigma)$

17. The value of c of Rolle's Theorem for the function $f(x) = \sin x \ \ in\ [0, \pi]$ is
    a. $\pi/3$
    b. $\pi/6$
    c. $\pi/2$
    d. None of the above

18. Linear and circular systematic sampling methods are equivalent if and only if:
    a. N (population size) is a whole number
    b. n (Sample size) is a whole number
    c. $N = n$
    d. None of the above

19. If $H_1 \& H_2$ are two subgroups of G, then the following is also always a subgroup
    a. $H_1 \cap H_2$
    b. $H_1 \cup H_2$
    c. $H_1 H_2$
    d. None of the above

20. If the sample values are 1, 3, 5, 7, 9 the standard error of sample mean is:
    a. $S.E = \sqrt{2}$
    b. $S.E = {}^{1}/_{\sqrt{2}}$
    c. $S.E = 2.0$
    d. $S.E = {}^{1}/_{2}$

21. Laplace transform of $\cos 2t + \sin 2t$ is
    a. $\frac{s+2}{s^2+4}$
    b. $\frac{s}{s^2+4}$
    c. $\frac{2}{s^2+4}$
    d. None of the above

22. An estimator is considered to be the best if its distribution is::
    a. Continuous
    b. Discrete
    c. Concentrated about the true parameter value
    d. Normal

23. The characteristic of a field is
    a. Zero
    b. A prime number
    c. Both a and b
    d. None of the above

24. Let X be a random sample of size one from a normal population with mean 0 and variance σ^2 . Then the sufficient statistics for σ^2 is:
    a. $|X|$

b. X
c. X^2
d. None of the above

25. The singular solution of the differential equation $y = xp + a\sqrt{1+p^2}$ is
    a. Parabola
    b. Hyperbola
    c. Circle
    d. Straight line

26. Crammer-Rao inequality with regard to the variance of an estimator provides:
    a. Upper bound on the variance
    b. lower bound on the variance
    c. asymptotic variance of an estimator
    d. None of the above

27. If $y_1 = e^{2x}$ and $y_2 = e^{3x}$, then the Wronskian $W(y_1, y_2)(x)$ at $x = 1/5$ is
    a. e
    b. 0
    c. 1
    d. e^2

28. If the density function of a variable X is
    $$f(x;\theta) = \theta e^{-\theta x} \quad for\ 0 < x < \infty,$$
    Then the 95% central confidence limits for large sample n are
    a. $(1 \pm \frac{1.96}{\sqrt{n}})\bar{x}$
    b. $(1 \pm \frac{1.96}{\sqrt{n}})/\bar{x}$
    c. $(\frac{1 \pm 1.96}{\sqrt{n}})/\bar{x}$
    d. None of the above

29. If the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and those of the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide, then what is the value of b^2
    a. 1
    b. 5
    c. 7
    d. 9

30. The Size of critical region is known as:
    a. Power of the test
    b. Size of type II error
    c. Critical value of the test statistics
    d. Size of the test

31. The value of the integral $\int_0^1 x^2(1-x^2)^{\frac{3}{2}}\,dx$ is
    a. $\pi/8$
    b. $\pi/4$
    c. $\pi/16$
    d. $\pi/32$

32. Degrees of freedom for Chi-square in case of contingency table of order (4×3) are :
    a. 12

b. 9
c. 6
d. 8

33. The system of equations $kx + y + z = 1,\ \ x + ky + z = k\ \ \&\ \ \ x + y + kz = k^3$ does not have a solution if k is equal to
    a. 0
    b. 1
    c. -1
    d. -2

34. Equality of several normal population means can be tested by:
    a. Bartlett's test
    b. F-test
    c. χ^2-test
    d. t-test

35. Let A be an nxn matrix from the set of real numbers and let $A^3 - 3A^2 + 4A - 6I = 0$ where I is the nxn unit matrix. If A^{-1} exists, then
    a. $A^{-1} = A - I$
    b. $A^{-1} = A + 6I$
    c. $A^{-1} = 3A - 6I$
    d. $A^{-1} = \frac{1}{6}(A^2 - 3A + 4I)$

36. Given the following eight sample values $-4, -3, -3, 0, 3, 3, 4, 4$, the value of student's- t to test $H_0: \mu = 0$ is :
    a. 2.73
    b. 0.97
    c. 3.30
    d. 0.41

37. Let M= $\begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 2 \\ 2 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$, then the rank of M is
    a. 3
    b. 4
    c. 2
    d. 1

38. If the two lines of regression are perpendicular to each other, the relation between the two regression coefficients is:
    a. $b_{yx} = b_{xy}$
    b. $b_{yx}.b_{xy} = 1$
    c. $b_{yx} \leq b_{xy}$
    d. $b_{yx} = -b_{xy}$

39. For the function $f(x) = \begin{cases} x & x\ rational \\ 1 - x & x\ irrational \end{cases}$ which one of the following statements is correct
    a. f is continuous at all rational numbers
    b. f is continuous only at $x = 0$

c. f is continuous only at $x = \frac{1}{2}$
d. f is not continuous at any point

40. If ρ is the correlation coefficient between X and Y, the correlation coefficient between $aX + b$ and Y is:
    a. $a\rho$
    b. $a\rho + b$
    c. $a^2\rho$
    d. ρ
41. The value of $\frac{5-i}{2-3i}$ is
    a. i
    b. $2i - 1$
    c. $1 + i$
    d. None of the above
42. Purchasing power of money is estimated by the formula:
    a. $\text{Price index} \times 100$
    b. $\frac{\text{Money income}}{\text{Consumer price index}} \times 100$
    c. $\frac{100}{\text{Price index}}$
    d. $\frac{\text{Price index}}{100}$
43. Every commutative division ring is
    a. An integral domain
    b. A field
    c. Both a and b
    d. None of the above
44. The ratio of births to the total deaths in a year is called :
    a. Survival rate
    b. Total fertility index
    c. Vital index
    d. Population death rate
45. The number of generators in a cyclic group of order 10 are
    a. 3
    b. 1
    c. 2
    d. 4
46. R-charts are preferable over σ-charts because:
    a. R and standard deviation fluctuate together in case of small samples
    b. R is easily calculable
    c. R-charts are economical
    d. All the above
47. The lines $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{3}$ & $\frac{x-4}{2} = \frac{y-6}{3} = \frac{z-7}{3}$ are coplanar. Their point of intersection is
    a. (4,6,7)
    b. (2,3,4)
    c. (1,1,1)
    d. (4,7,10)

48. In a sequential sampling plan, the sample size is :
    a. A discrete random variable
    b. A continuous random variable
    c. Both a and b
    d. Neither a nor b

49. The union of an arbitrary family of closed sets is always
    a. Closed
    b. Open
    c. Neither open no closed
    d. None of the above

50. For consumer price index, price quotations are collected from
    a. retailers
    b. wholesale dealers
    c. fair price shops
    d. government depots

51. The modulus of $\frac{2i}{(1+i)(1-i)}$ is
    a. 2
    b. 1
    c. $\frac{1}{2}$
    d. None of these

52. Which of the following is contrast :
    a. $3T_1 + T_2 - 3T_3 + T_4$
    b. $T_1 + 3T_2 - 3T_3 + T_4$
    c. $-3T_1 - T_2 + T_3 + 3T_4$
    d. $T_1 + T_2 + T_3 - T_4$

53. If A and B are two matrices such that AB=B and BA=A, then $A^2 + B^2 =$
    a. 2AB
    b. 2BA
    c. A+B
    d. A-B

54. The Newton's- Gauss backward formula is to be applied when:
    a. The interpolating value of argument lies in the middle of the series
    b. The interpolating value of argument lies at the end of the series
    c. The interpolating value of argument lies anywhere in the series
    d. None of the above

55. If $\vec{i}, \vec{j}, \vec{k}$ have the usual meaning in vectors, then $\vec{i}.\vec{j} = \vec{j}.\vec{k} = \vec{k}.\vec{i} =$
    a. -1
    b. 0
    c. 1
    d. 2

56. The production of lignite in India from 1975 to 1985 in Mn. Tones was,
    3.03, 4.02, 3.58, 3.30, 2.90, 5.11, 6.31, 6.93, 7.30, 7.80, 8.03

It is expected that the median production of lignite in India is 5 Mn. Tones/year. To test $H_0: M = 5.0$, the value of T^- in Wilcoxon signed rank test is:

a. 28
b. 27
c. 26
d. 25

57. $\lim_{n\to\infty} \frac{1}{\sqrt{n}} \left(\frac{1}{\sqrt{1}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \ \ldots \ \frac{1}{\sqrt{2n-1}+\sqrt{2n+1}} \right)$ equals

a. $\sqrt{2}$
b. $\frac{1}{\sqrt{2}}$
c. $1+\sqrt{2}$
d. None of the above

58. The odds in favor of certain event are $5:4$, and odds against another event are $4:3$. The chance that at least one of them will happen is:

a. $15/63$
b. $51/63$
c. $47/63$
d. None of the above

59. If one root of $x^2 - x - \lambda = 0$ is square root of the other, then the value of λ is

a. $2 \pm \sqrt{3}$
b. $2 \pm \sqrt{5}$
c. $5 \pm \sqrt{2}$
d. $5 \pm \sqrt{3}$

60. The missing value for the following data,

x:	5	10	15	20
y:	2	5	?	8

by the binomial expansion method is:

a. 7
b. -7
c. 3
d. 25/3

Entrance Test Answer Key – Actuarial and Financial Mathematics, 2010

Section A

1. b
2. a
3. b
4. c
5. b

Section B

1. a
2. b
3. c
4. a
5. a
6. d
7. b
8. b
9. d
10. c
11. c
12. d
13. a
14. b
15. a
16. b
17. c
18. d
19. a
20. a
21. a
22. c
23. c
24. c
25. b
26. b
27. a
28. b
29. c
30. d
31. d
32. c
33. d
34. b
35. d
36. d
37. c
38. d
39. c
40. d
41. c
42. c

43. b
44. c
45. a
46. d
47. b
48. a
49. d
50. a
51. b
52. c
53. c
54. b
55. b
56. c
57. b
58. c
59. b
60. a